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# C. U. SHAH UNIVERSITY Summer Examination-2022 

## Subject Name : Numerical Methods

Subject Code: 4SC04NUM1
Semester: 4

Date: 06/05/2022

Branch: B.Sc. (Mathematics)
Time: 11:00 To 02:00 Marks: 70

Instructions:
(1) Use of Programmable calculator \& any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

## Q-1 Attempt the following questions.

a) The interval in which the root of the equation $x^{3}-x-11=0$ lies is $\qquad$ .
i) $\quad(0,1)$
ii) $(2,3)$
iii) $(1,2)$
iv) None of these
b) The order of convergence in Newton's Raphson method is $\qquad$ .
i) 2
ii) 3
iii) 0
iv) None of these
c) In Simpson's $\frac{1}{3}^{\text {rd }}$ rule,$n$ is multiple of $\qquad$ .
i) 2
ii) 3
iii) 4
iv) 5
d) Which of the following method can be used to evaluate a numerical integral?
i) Euler's Modified Method
ii) Picard's Method
iii) Runge-Kutta's Method
iv) Taylor Series Method
e) Newton's Iterative formula to find the value of $\sqrt{N}$ is $\qquad$ .

i) $\quad x_{n+1}=x_{n}+\frac{N}{x_{n}}$
ii) $\quad x_{n+1}=\frac{1}{2}\left(x_{n}+\frac{N}{x_{n}}\right)$
iii) $\quad x_{n+1}=\frac{1}{2}\left(x_{n}-\frac{N}{x_{n}}\right)$
iv) $\quad x_{n+1}=\frac{1}{3}\left(2 x_{n}+\frac{N}{x_{n}^{2}}\right)$
f) Varies type of Runge-Kutta methods are classified according to their $\qquad$ .
i) degree
ii) order
iii) rank
iv) none of these
g) True/False : The second order Runge-Kutta formula is Euler's method.
h) True/False : Newton Raphson method is applicable to the solution of both algebraic and transcendental method.
i) Write Milne's corrector formula.
j) Which formula we use to derive Trapezoidal rule ?
k) Write formula of Euler's Modified method.
l) Write Newton's Iterative formula to find value of $\sqrt[3]{\mathrm{N}}$.

## Attempt any four questions from Q-2 to Q-8

a) Derive General Quadrature formula.
b) Find area bounded by curve and $x$-axisfrom the following table from
$x=7.47$ to $x=7.52$

| $x$ | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 1.93 | 1.95 | 1.98 | 2.01 | 2.03 | 2.06 |

## Q-3 Attempt all questions.

a) Find the $1^{\text {st }}$ and2 ${ }^{\text {nd }}$ order derivatives of $f(x)$ at $x=1.05$ and $x=1.25$

| $x$ | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 1.00 | 1.02470 | 1.04881 | 1.07238 | 1.90544 | 1.11803 | 1.14017 |

b) Obtain $f^{\prime}(90)$ using Sterling's formula from the following table:

| $x$ | 60 | 75 | 90 | 105 | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

## Q-4 Attempt all questions.

a) Derive differentiation formula based on Newton's forward interpolation formula.
b) Derive Euler Maclaurin Sum formula.

## Q-5 Attempt all questions.

a) Find a real root of $\sin x=1+x^{3}$, correct to three decimal places using

Newton Raphson method.
b) Evaluate $\int_{0}^{6} \frac{d x}{1+x^{2}}$ by Simpson's $\frac{1}{3}^{\text {rd }}$ Rule. Take $h=1$.
c) Find a positive root of the equation $x e^{x}=1$ which lies between 0 and

1 , correct to three decimal places using Bisection method.

## Q-6 Attempt all questions.

a) Evaluate $\int_{0}^{5} \frac{d x}{1+x}$ by Trapezoidal Rule.Take $h=1$.
b) Find a real root of the equation $2 x-\log _{10} x=7$, correct to three decimal places using Iteration method.
c) Find a real root of the equation $x e^{x}-2=0$ which lies between 0.8 and 0.9 ,correct to three decimal places using Regula-falsi method.

## Q-7 Attempt all questions.

a) Use Taylor series method to obtain approximate value of $y$ at $x=$ 0.2 provided $\frac{d y}{d x}=2 y+3 e^{x}, y(0)=0$.Compare the numerical solution with exact solution.
b) Given $\frac{d y}{d x}=y-x$ where $y(0)=2$.Find $y(0.1)$ and $y(0.2)$ by Runge Kutta method for $3^{r d}$ order, correct upto four decimal places.

Q-8 Attempt all questions.
a) Use Picard's method to compute $y(0.1)$ from the differential equation
$\frac{d y}{d x}=x+y, y(0)=1$.
b) Given $\frac{d y}{d x}=x^{3}+y, y(0)=1$.Find $y(0.02)$ by Euler's method .Correct up to four decimal places. Take $h=0.01$

